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# RATIONAL STRUCTURE OF BLOOD VESSELS

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*The issue of the optimal (from the viewpoint of strength) structure of blood vessels of a living organism is considered. It is shown that the angle of packing of muscular fibers in vessels is optimal in terms of strength of arteries.*

**Key words:** *blood vessels, reinforced vessels, shell, composite, viscosity-elasticity, deformation.*

Based on extensive clinical research [1], blood vessels (arteries and arterioles) can be presented as a multilayer shell. Smooth muscles in large blood vessels are known to be aligned at an angle $\varphi = 30$–$50^\circ$ to the vessel centerline. The angle $\varphi$ increases with decreasing blood-vessel diameter (Fig. 1), i.e., in moving downward in the circulatory system from arteries to arterioles and further to capillaries. The angle of inclination of smooth muscles in small blood vessels approaches $90^\circ$.

Figure 2 shows the general structure of arteries and arterioles. They consist of an inner layer (*tunica interna*), medium layer (*tunica media*), and outer layer (*tunica externa*). The inner layer consists of endotherlium and an internal elastic membrane. The medium layer consists of smooth-muscle cells and a certain amount of fibroplastic elements and collagen fibers. The number of fibroplastic elements and collagen fibers varies in different arteries (elastic, muscular-elastic, and muscular types). There are few elastic fibers in arterioles, but precapillary sphincters are present. The density of muscle cells in the arteriole walls depends on the distance between the arteriole and the parent small artery. The outer layer consists of an external elastic membrane and a loose fibrous connective tissue.

Let us consider the medium layer of the blood vessel in more detail. The smooth-muscle cells and fibers form an elastomotor helix (see [2]) inclined to the vessel centerline. The angle between the elastomotor helix winding and the longitudinal axis of the vessel is 30–50° in large arteries, gradually increasing as the vessel diameter decreases. In small arterioles, the winding direction is closer to circular [1–5]. With an increase in years, the number of layers of the helical winding of muscular fibers increases from three layers (at the age of 12) to six layers (at the age of 20) [2].

Elastic properties of vascular walls play a significant role in oscillatory motion of blood. The elastic state of artery walls is not stable. It changes reflexively due to activity of muscular elements, which are nonuniformly distributed in various parts of the arterial network. Depending on organism demands (level of blood circulation), the elastic properties of vascular walls can vary. Therefore, the modulus of elasticity of vascular walls depends on the functional state of muscular elements and on the degree of passive stress of the connective tissue of the vessel.

Let us consider the blood-vessel wall (Fig. 3) as a binder with embedded muscular elements (reinforcing fibers).

Let the volume fraction of fibers be $\omega_s$. The family of reinforcing fibers is aligned at an angle $\varphi$ to the vessel centerline (dashed straight line in Fig. 3). The volume fraction of the binder $\omega_0$ is

$$\omega_0 = 1 - \omega_s. \qquad (1)$$

The family of reinforcing fibers is assumed to be an elastic material with Young's modulus $E_s$. Under the loads applied, all fibers are assumed to retain elasticity. The binder is considered as a linearly viscoelastic material with Young's modulus $E_0$ and Poisson's ratio $\nu$.

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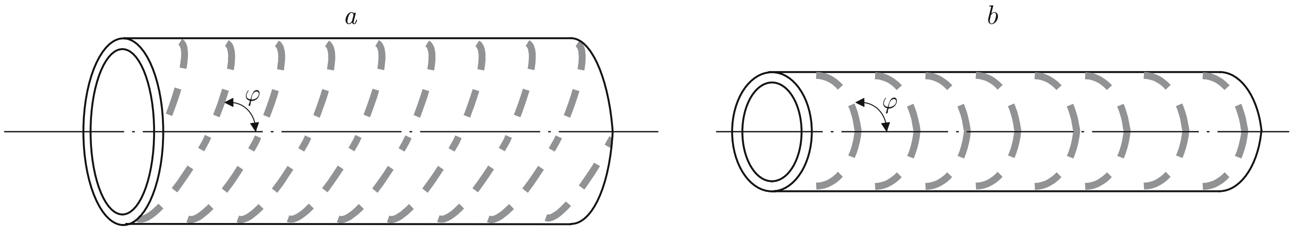


Fig. 1. Inclination of smooth muscles in large (a) and small (b) blood vessels.

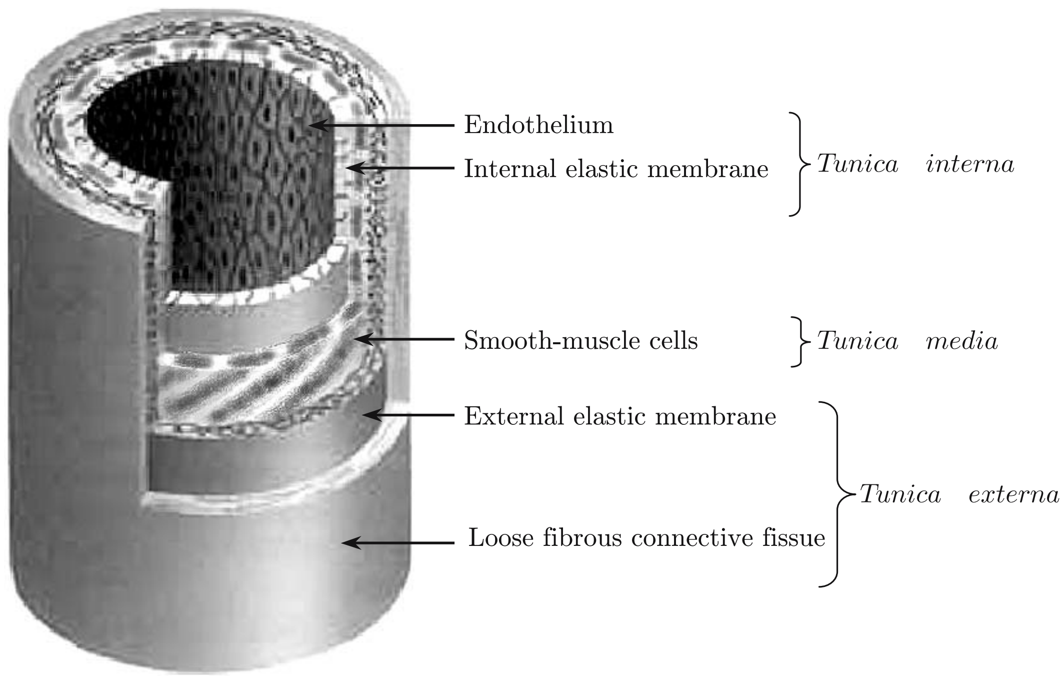


Fig. 2. Structure of an artery.

The numerical model of deformation of a cylindrical layered vessel (shell) should include three groups of equations: 1) equations of equilibrium or motion of the shell element; 2) geometric relations derived on the basis of an assumption about the deformation character, which are essentially related to the geometric parameters of deformation; 3) physical expressions relating stresses and strains and describing the material properties. The second and third groups of equations describe the specific features of mechanics of deformation of a composite material that is implied to be the blood-vessel material.

On the basis of Timoshenko's generic kinematic hypothesis [6] being used, the resolving equations can be presented as [7]

$$(A-B)\boldsymbol{u}+\tilde{h}^2\boldsymbol{p}=0; \tag{2}$$

$$\boldsymbol{Q}\cdot\delta\boldsymbol{u}\Big|_{\Gamma}=0, \tag{3}$$

where $A$ is the kinematic matrix-operator, $B$ is the inertial matrix-operator, $\boldsymbol{u}$ and $\boldsymbol{p}$ are the vectors of generalized displacements and external surface load, $\boldsymbol{Q}$ and $\delta\boldsymbol{u}$ are the vectors of generalized forces and variations of generalized displacements on the line $\Gamma$ bounding the reference surface (mid-surface) of the shell, and $\tilde{h}$ is the shell thinness (defined below).

The boundary-value problem (2), (3) is nonlinear in the general case. The total order of the system is 12 and does not depend on the number of layers and their positioning. In contrast to classical models described, e.g., in [6], shear strains and stresses here are determined naturally in terms of appropriate elasticity relations [8].

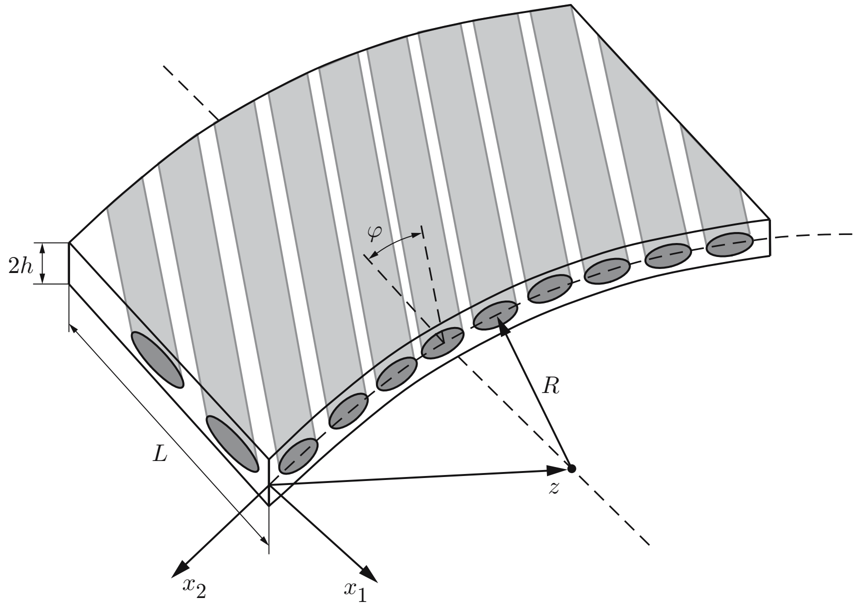


Fig. 3. Schematic of the blood-vessel wall: $L$ is the vessel length, $h$ is the half-width of the vessel wall, and $R$ is the vessel radius; the gray regions are the muscular (reinforcing) fibers.

The critical load (maximum internal pressure that the vessel can sustain) obeys the dependence [8]

$$\tilde{q} = \frac{\beta}{12}\varphi_1 + \frac{\theta^4}{\beta\varphi_2}, \tag{4}$$

where $\varphi_1 = a_{11}\theta^2 + 2(a_{12} + 2a_{33}) + a_{22}/\theta^2$, $\varphi_2 = A_{22}\theta^2 + 2A_{12} + A_{33} + A_{11}/\theta^2$, and $\mathcal{E} = [a_{ij}]$ $(i, j = 1, 2, 3)$ is a symmetric matrix of elasticity coefficients of the binder and fibers in a planar stress state.

In the case of an isotropic viscoelastic binder, the coefficients of the matrix $\mathcal{E}$ have the form

$$\begin{gathered} a_{ii} = \frac{1}{1-\nu^2} + P_s\chi_i^4, \qquad a_{12} = a_{21} = \frac{\nu}{1-\nu^2} + P_s\chi_1^2\chi_2^2, \qquad a_{13} = a_{31} = P_s\chi_2\chi_1, \\ a_{23} = a_{32} = P_s\chi_1\chi_2, \qquad a_{33} = \frac{1}{2(1+\nu)} + P_s\chi_1^2\chi_2^2, \end{gathered} \tag{5}$$

where $P_s = \omega_s\tilde{E}_s/\omega_0$ and $\chi_i = \text{const}$ is a direction cosine of the trajectory of the fiber family with respect to the direction $i$. For the coordinate system shown in Fig. 3, the direction cosines are $\chi_1 = \chi$, $\chi_2 = \sqrt{1-\chi^2}$, and $\chi_3 = 0$ $(\chi = \cos\varphi)$. The matrix is $\|A_{ij}\| = \|a_{ij}\|^{-1}$ $(i, j = 1,\ 2,\ 3)$.

Formulas (4), (5) involve the following dimensionless variables: $\tilde{q} = q/(4\omega_0 E_0\tilde{h}^2)$ ($q$ is the dimensional internal pressure acting on the vessel), $\tilde{h} = h/R$, $\tilde{E}_s = E_s/E_0$, $\tilde{R} = R/L$ (dimensionless radius of the vessel), $\beta = 2\tilde{h}\pi^2\tilde{R}^2$, and $\theta = \pi\tilde{R}/n$ ($n$ is the number of waves characterizing the shape of circumferential deformation of the vessel). In what follows, we use $n = 2$, which means that the vessel is deformed to an oval.

Equation (4) was derived for solving the problem of stability under external pressure. It can also be used, however, to determine the internal pressure of a cylindrical vessel if we use $n = 0$ or $n = 2$ and $m = 1$ ($m$ is the number of waves formed along the vessel generatrix). This is valid in solving the problem of stability by the Bubnov–Galerkin method [8] in a linearized formulation [6].

Equation (4) defines the maximum internal pressure that the vessel can sustain as a function of the angle $\varphi$ of reinforcing fibers (Fig. 4). It is seen from Fig. 4 that the maximum internal pressure that the vessel can sustain corresponds to an angle of packing of muscular fibers equal to $\varphi \approx 57°$.

To study the properties of packing of smooth muscles, we have to determine the elastic characteristics of blood vessels. This is rather difficult because the mechanical properties of vessels are not constant: they change with ageing and under the action of various medications. A comprehensive review on viscoelastic properties of artery walls can be found in [9].

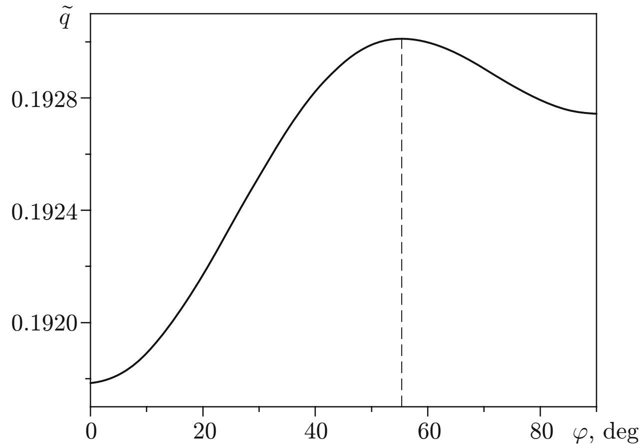


Fig. 4. Maximum internal pressure versus the angle of fibers ($\tilde{R} = 0.32$, $\tilde{h} = 0.09$, $\tilde{E}_s = 0.03$, $\nu = 0.5$, and $\omega_s = 0.1$).

According to [9], Poisson's ratio $\nu$ of blood-vessel walls can be assumed to equal 0.5 (this does not introduce any significant errors in computations).

The muscular shell of the vessels performs two kinds of work: static (overcoming intravascular blood pressure) and dynamic (active screwed laminar pushing of blood in the vessel) [2, 10]. Obviously, the ultimate values of blood-vessel strength are determined under peak static loads. It was found [11] that the walls of the dog's common carotid artery sustains an internal (outward) pressure up to $88 \cdot 10^6$ dyn/cm$^2$. When only the muscular shell of this artery was inspected (the external and internal membranes were removed), it turned out that the muscular shell sustains a pressure of $2.73 \cdot 10^6$ dyn/cm$^2$ only.

Let us consider a homogeneous elastic cylindrical shell under the action of internal pressure. The maximum pressure that a homogeneous shell can sustain is (see [6])

$$p_{\max} = \frac{\pi\sqrt{6}}{9(1-\nu^2)^{0.75}} E \frac{h^2}{RL} \sqrt{\frac{h}{R}}. \qquad (6)$$

To estimate Young's modulus $\tilde{E}_s = E_s/E_0$, we use the ratio of the ultimate pressure for the muscular shell to the ultimate pressure of the common carotid artery. Then, according to Eq. (4), the ratio of Young's moduli of the reinforcing fibers $E_s$ and the binder $E_0$ can be estimated as $\tilde{E}_s = 0.03$.

The ratio of the vessel-wall thickness to the vessel radius is fairly constant [9]. Therefore, the quantity $D = (r_a - r_i)/r_i$ ($r_a$ and $r_i$ are the outer and inner radii of the artery, respectively) can be considered as a constant without large errors. According to [9], $D = 0.15$ for arteries of the elastic type and $D = 0.20$ for arteries of the muscular type. Then the value of $\tilde{h}$ varies from 0.07 (elastic-type arteries) to 0.09 (muscular-type arteries).

The most difficult task is to estimate the volume fraction of muscular fibers $\omega_s$ in the blood-vessel wall. Obviously, this fraction depends on the artery type (elastic, muscular-elastic, or muscular) and on the age of the living organism (as was noted above). Results for $\omega_s = 0.1$ are presented below. The optimal angle of inclination of muscular elements is independent of the volume fraction of muscular fibers $\omega_s$.

Equation (4) also yields an optimal angle of packing of fibers $\varphi_{\rm opt}$, which is an angle at which the vessel can sustain the maximum pressure. This dependence is found by solving the equation

$$\frac{\partial \tilde{q}(\varphi; \tilde{R}, \tilde{E}_s, \nu, \omega_s, \tilde{h})}{\partial \varphi} = 0, \qquad (7)$$

which yields the optimal angle of packing of muscular fibers

$$\varphi_{\rm opt} = \varphi(\tilde{R}; \tilde{E}_s, \nu, \omega_s, \tilde{h}). \qquad (8)$$

The optimal angle of muscular fibers $\varphi_{\rm opt}$ (8) is plotted in Fig. 5 as a function of the vessel radius $\tilde{R}$. It is seen that the angle of inclination of muscular fibers for large blood vessels ($\tilde{R} = 0.5$) is close to 43°. As the vessel radius decreases to $\tilde{R} = 0.3$, the angle of packing of muscular fibers increases to 83° for muscular-type arteries ($\tilde{h} = 0.09$)

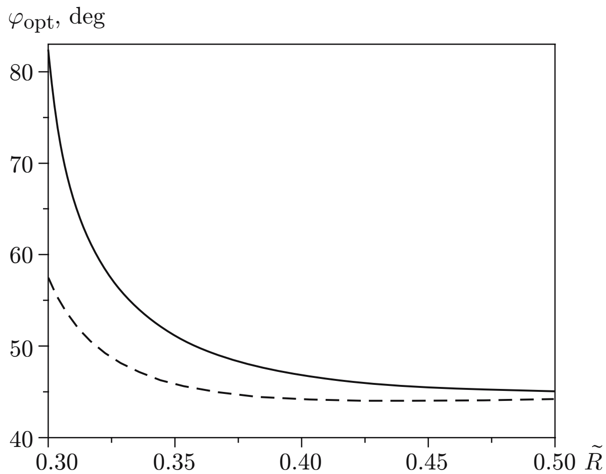


Fig. 5

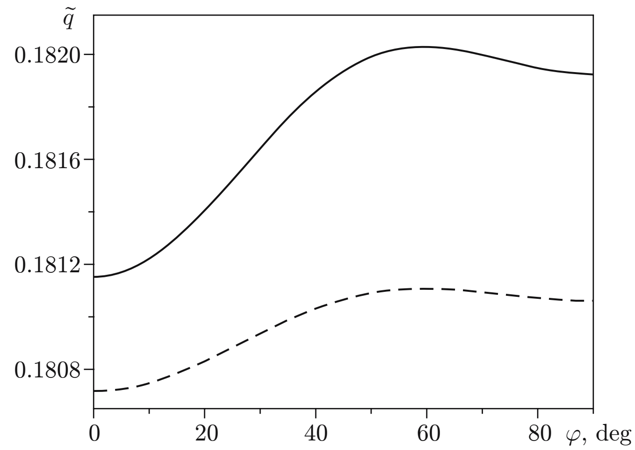


Fig. 6

Fig. 5. Optimal angle of packing of muscular fibers versus the blood-vessel radius ($\tilde{E}_s = 0.03$, $\nu = 0.5$, and $\omega_s = 0.1$): solid and dashed curves refer to $\tilde{h} = 0.09$ and 0.07, respectively.

Fig. 6. Effect of "ageing" of the vessel on its optimal strength ($\tilde{R} = 0.32$, $\tilde{h} = 0.09$, $\tilde{E}_s = 0.01$, and $\nu = 0.5$): solid and dashed curves refer to $\omega_s = 0.2$ and 0.1, respectively.

and to 58° for elastic-type arteries ($\tilde{h} = 0.07$). This dependence of the angle of smooth-muscle elements on the vessel size is in agreement with morphological data [2] on orientation of muscles in artery walls.

Let us study the effect of "ageing" of the vessel on its optimal strength. The maximum internal pressure is plotted in Fig. 6 as a function of the angle of packing of fibers. The dashed curve in Fig. 6 shows the results calculated for "ageing" of a vessel with an unchanged volume of muscles ($\omega_s = 0.1$). Elasticity of muscular fibers $\tilde{E}_s$ is only one third of that calculated for a young organism (cf. the solid curves in Figs. 4 and 6), i.e., the strength of the vessel decreases. The loss of strength can be partly compensated by a twofold increase in the muscle bulk ($\omega_s = 0.2$) (solid curve in Fig. 6). The strength of the vessels in a young organism, however, cannot be reached again (cf. Figs. 4 and 6). The preliminary results of this work are described in [12, 13].

Thus, it is shown that smooth muscles in large blood vessels (arteries and arterioles) are wound in an optimal manner. "Ageing" of vessels (decrease in elasticity of their walls) in living organisms is compensated by an increase in muscle bulk. Depending on the vessel size, muscular fibers are aligned at an angle providing the maximum possible strength of the vessel. This proves again that the structure of the living organism (in particular, circulatory system) is optimal from the viewpoint of mechanics.

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